

Understanding the Significance, Processing & Analysis of Point Clouds

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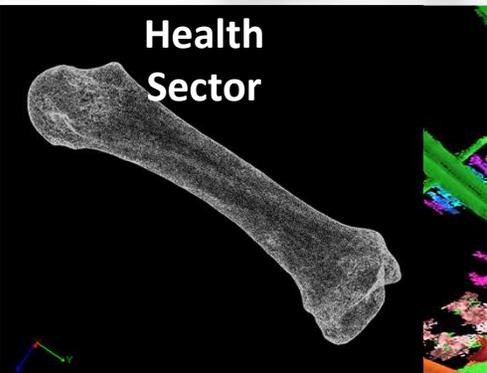
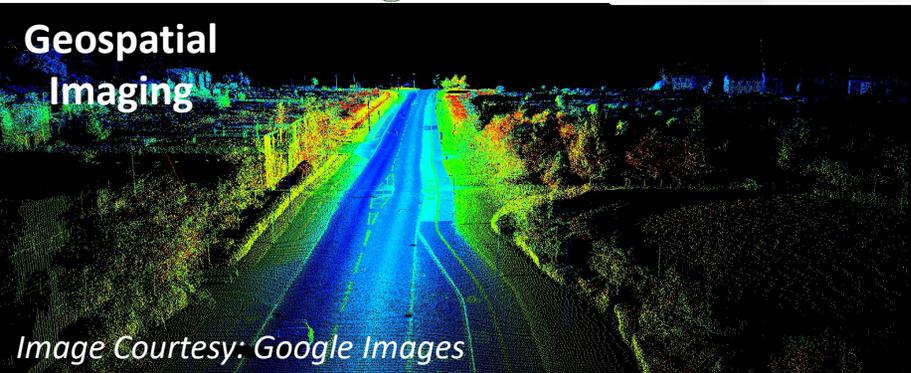
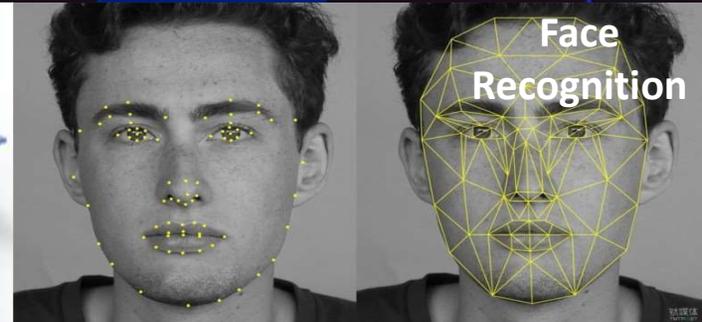
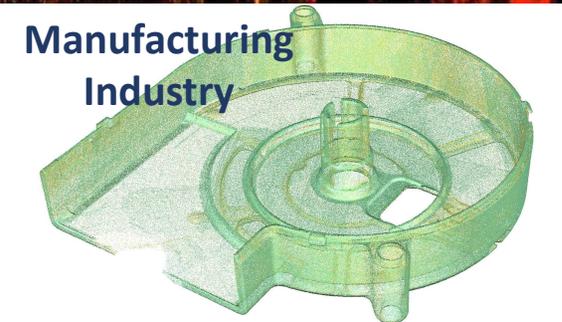
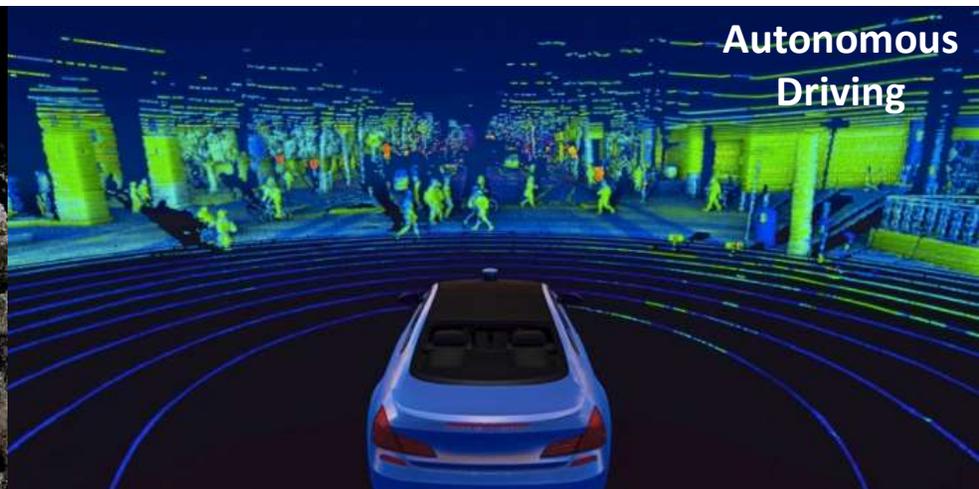


Image Courtesy: Google Images

Structure Tensor-Based Geometric Features of Point Clouds

- For a point cloud P , if N_{p_i} is the set of k -nearest neighbours of a point $p_i \in P$, then the Covariance Matrix(C) or Structure Tensor(S) is defined as:

$$C = S = \begin{bmatrix} p_{i_1} - p_i \\ p_{i_2} - p_i \\ \cdot \\ \cdot \\ p_{i_k} - p_i \end{bmatrix}^T \cdot \begin{bmatrix} p_{i_1} - p_i \\ p_{i_2} - p_i \\ \cdot \\ \cdot \\ p_{i_k} - p_i \end{bmatrix}$$

where $p_{i_j} \in N_{p_i}$.

- Spherical or cylindrical neighbourhoods of fixed radius centered at point p_i are also used.
- Here, $\lambda_1 > \lambda_2 > \lambda_3$.

Geometric Feature	Expression	Significance
Linearity	$\frac{\lambda_1 - \lambda_2}{\lambda_1}$	Describes how well the points constitute a line.
Planarity	$\frac{\lambda_2 - \lambda_3}{\lambda_1}$	Describes how well the points constitute a plane.
Sphericity or Scattering	$\frac{\lambda_3}{\lambda_1}$	Describes how well the points constitute a sphere.
Omnivariance	$\sqrt[3]{\lambda_1 \cdot \lambda_2 \cdot \lambda_3}$	Describes the average point density in all directions.
Anisotropy	$\frac{\lambda_1 - \lambda_3}{\lambda_1}$	Describes whether the points are distributed in a specific direction, or if they are randomly distributed.
Change of Curvature or Surface Variation	$\frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$	Describes the degree of bending of a curved line or a plane and is the derivative of the curvature.

Table 1. Description of Geometric Features based on the eigenvalues of the Structure Tensor

Point Cloud and Mesh Simplification

- Definition: Given a collection of 3D data points (or an initial triangular mesh), sample a subset of points (or find a new mesh with a smaller number of vertices) such that both the overall shape and the most salient features are preserved in the simplified point cloud (or mesh).
- Significance: Huge amount of redundant data produced by point cloud capturing devices makes the processing, transmission and storage of data extremely expensive.
- Most of the existing techniques either take meshes as input or triangulate the point cloud as a pre-processing step.

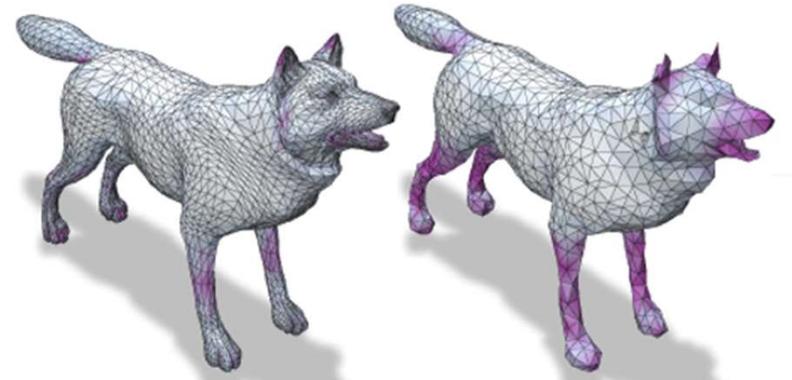


Figure 1. Mesh Simplification

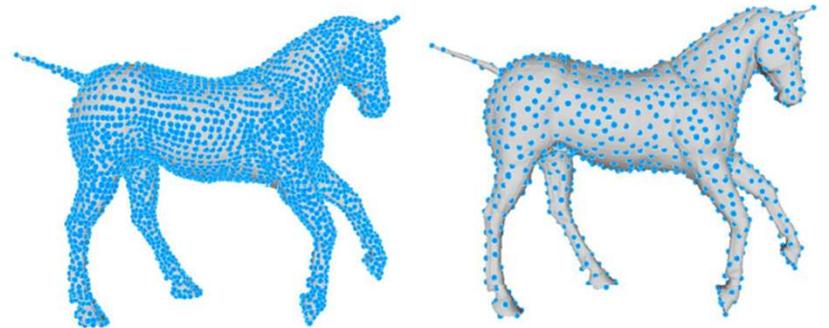


Figure 2. Point Cloud Simplification

Mesh Decimation

- Characterize all vertices.

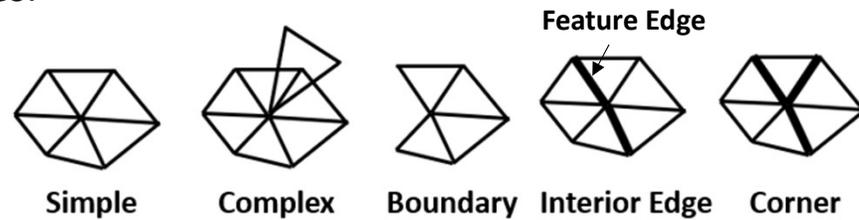


Figure 3. Vertex Classification

- Assign a distance criterion to each vertex type which determines the potential deletion candidacy.

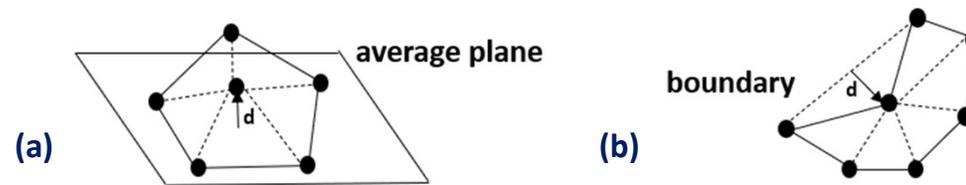


Figure 4. (a) Distance to face, (b) Distance to edge

- Multiple passes are made through the whole mesh. In each pass:
 - a vertex and its edges are removed.
 - the newly formed loop is re-triangulated.

Energy Function Minimization

- Minimize the energy function E :

$$E = E_{dist} + E_{rep} + E_{spring}$$

where:

- E_{dist} is the sum of squared distances from the points to the mesh.
 - E_{rep} is kept proportional to number of points (m).
 - E_{spring} places a spring on each edge of the mesh.
- $E_{dist} \uparrow$ and $E_{rep} \downarrow$ when $m \downarrow$. So both work together and penalize each other to minimize E .
 - A minimum of $E_{dist} + E_{rep}$ may not exist which is solved by the introduction of E_{spring} .

Coplanar Merging

- Group all nearly coplanar polygons.
- Create edge list.
- Remove redundant edges and keep only boundary edges.
- Reconstruct polygons.
- Re-triangulate to deal with holes.

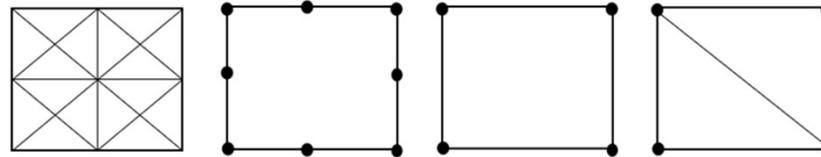


Figure 5. Overview of Simplification Process

Vertex Clustering

- Assign weights to all vertices based on their importance.
- Triangulate all the vertices.
- Divide the object into cells such that all vertices in the same cell form a cluster.
- Synthesis: Assign each cluster a single vertex which is defined by COM of all the weighted cluster vertices.
- Elimination: Triangles with three same representative vertices are replaced by points and ones with only two are replaced by an edge.

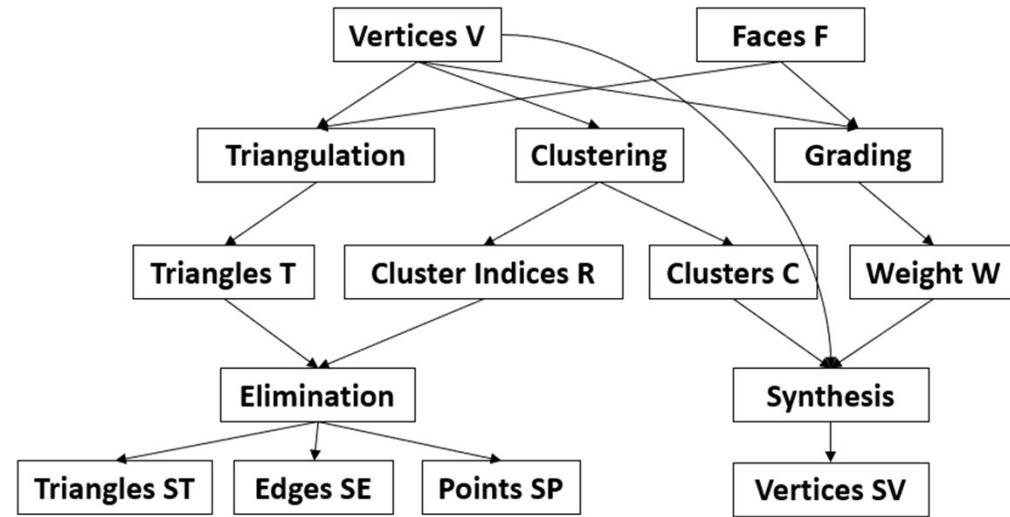


Figure 6. Overview of Simplification Process

Re-Tiling

- Construct a triangular mesh from the original vertices.
- Choose a set of new candidate vertices that lie in the planes of the original mesh and may coincide with the original vertices.
- Move each point away from all nearby points by a relaxation method.
- Mutual tessellation: Create triangular meshes using both original and new vertices.
- One by one remove original vertices and triangulate to maintain the original topology.

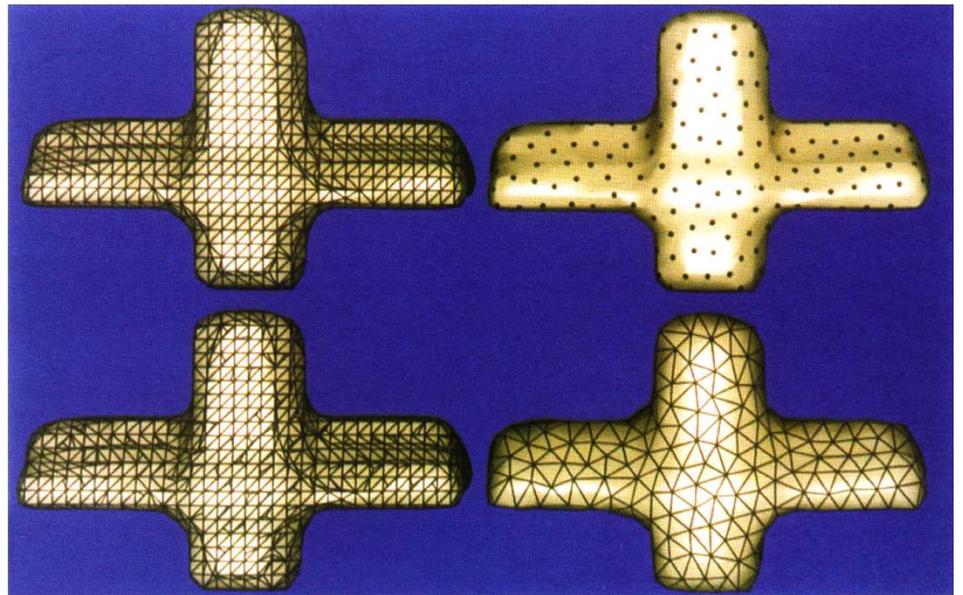


Figure 7. Overview of Simplification Process

Graph Neural Network-Based Simplification

- A point-wise MLP projects all the points to a latent space.
- A GNN (optionally with mesh edges) captures the local geometric features.
- Use Farthest Point Sampling (FPS) to find cluster centers in latent space.
- Each cluster center is connected to all of its k-nearest neighbours by edges.
- A final layer updates the cluster center positions based on the neighbours to minimize visual perceptual error and preserve salient features.

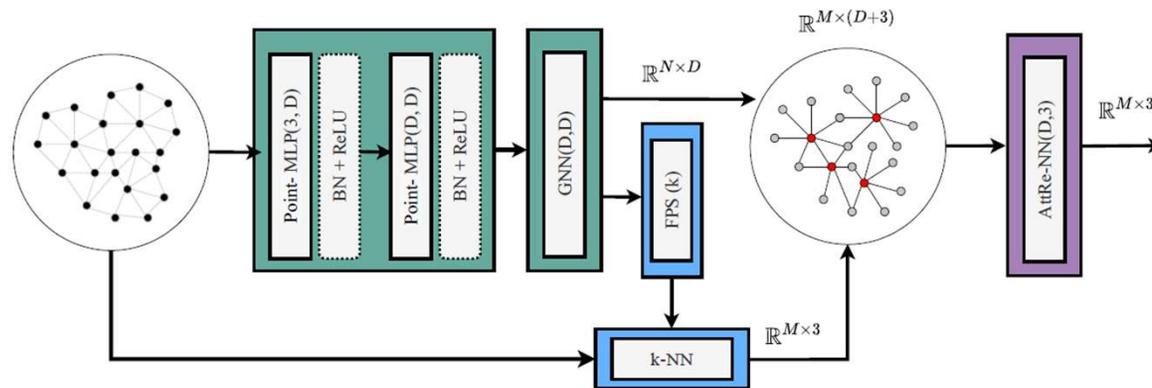


Figure 8. Overview of Simplification Process

Laplace-Beltrami Operator for Point Clouds

- For a point cloud P , if A_i is the Voronoi weight at point $p_i \in P$, then the Laplace Operator (L) is defined as:

$$L[i][j] = \begin{cases} G_h(i, j)A_j & i \neq j \\ G_h(i, i)A_i - \sum_{j=1}^n [G_h(i, j)A_j] & \text{otherwise} \end{cases}$$

where $G_h(i, j) = \frac{1}{4\pi h^2} e^{-\frac{\|p_i - p_j\|^2}{4h}}$

- In weighted Voronoi diagram, weights are assigned to each point based on their importance. These weights determine the size of a point's Voronoi cell.
- Since L is not symmetric, it is decomposed into $L = GD$ where D is a diagonal matrix with $D[i][i] = A_i$ and $G[i][j] = G_h(i, j)A_j$ for $i \neq j$ and $G[i][j] = -\frac{1}{A_j} \sum_{j \neq i} G_h(i, j)A_j$ otherwise.
- Eigenvalues $\lambda_1, \dots, \lambda_n$ and eigenvectors ϕ_1, \dots, ϕ_n of L are computed by solving generalized eigenvalue problem $G\rho_i = \beta_i D^{-1}\rho_i$ where $\lambda_i = \beta_i$ and $\phi_i = D^{-1}\rho_i$ as $GD(D^{-1}\rho_i) = G\rho_i = \beta_i(D^{-1}\rho_i)$.

Role of Gaussian Processes

- Terrain Modelling: GPs can successfully model large-scale terrains and preserve the spatial features efficiently.
- Object Segmentation: GPs provide enhanced segmentation accuracy by reducing over-segmentation.
- Free Space Detection: GPs are used to fuse multimodal information (2D from camera and 3D data from Lidar) from autonomous vehicles to aid detection of space available to freely transverse.
- Scene Categorization: Multi-class GP classification have been used to categorize outdoor scenes with informative uncertainty estimates as an added value.
- Object tracking and Shape Detection: GPs can efficiently track the dynamic behaviour of objects represented by sparse point clouds along with estimating the position, orientation and shape of the object.
- Point cloud registration: GPs have been used to extract keypoints in multiple sets of point clouds for further aligning and merging into a globally consistent model.

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Thank you!